

# On supersymmetry breaking three-form flux on Sasaki-Einstein manifolds

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## Abstract

Studying nonsupersymmetric yet imaginary self-dual three-form fluxes in type IIB supergravity backgrounds on Sasaki-Einstein manifolds we find a new analytic solution that restores supersymmetry in the IR, breaks it at higher energies, yet suffers from curvature singularities in the UV, when a certain SUSY-breaking parameter becomes large.

Consequently we investigate a variety of possibilities to cure the singularity by either introducing additional sources, changing the fluxes or deforming the geometry. Since it is not possible to cure the singularities making physically reasonable assumptions, we suggest that there might be a no-go theorem disallowing such flux.

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## I. INTRODUCTION

Progress in gauge/string duality is considerably helped by supersymmetry in various guises: The construction and deformation of supergravity backgrounds is often based on the study of the relevant BPS-equations. D-branes and their backreaction can be addressed using  $\kappa$ -symmetry or generalized calibrations. Four-dimensional geometries preserving four supercharges can be described in terms of generalized geometry. Finally, the stability of both the backgrounds and embedded branes follows directly from the existence of conserved supercharges. Yet albeit the uses of supersymmetry, there is an obvious interest in gauge/string duality for non-supersymmetric solutions.

Of course, the problem is not a new one and there have been a number of different approaches. Examples of explicit supersymmetry-breaking can be found in [1] and the Sakai-Sugimoto model [2], which are both based on D4-branes wrapping an  $S^1$ . Supersymmetry is broken due to antiperiodic boundary conditions for the fermions on the  $S^1$ . Conversely, models such as [3] and [4] are based on deformations of  $\text{AdS}_5 \times S^5$ . By giving a radial profile to various scalar functions in either five- or ten-dimensional supergravity they allow in principle for both supersymmetric and non-supersymmetric solutions. Another approach is to study linearized perturbations around a known supersymmetric solutions, as was done in [5] for the Maldacena-Núñez background and in [6] and [7] for the Klebanov-Strassler solution. Further progress has been made in the context of the latter. Here, [8] is concerned with a modification of the calibration condition for D7-branes, while there has been very recent progress towards incorporating the backreaction of anti-branes, see e.g. [9].

The purpose of this note is to investigate the possibility of following a different approach. The various conifold theories (see [10], [11] and [12]), their generalizations to metric cones over Sasaki-Einstein manifolds ([13], [14]) as well as the Maldacena-Núñez background [15] are all examples of  $SU(3)$ -structure backgrounds in type IIB string theory. Hence one can ask whether it is possible to exploit the existence of the  $G$ -structure when breaking the supersymmetry in these solutions.

In the context of string compactifications this question has been addressed by the Domain Wall Supersymmetry Breaking (DWSB) mechanism of [16]. The idea is as follows. Warped supergravity backgrounds of the form  $\mathbb{R}^{1,3} \times_A \mathcal{M}_6$  preserving four supercharges can be described in terms of generalized geometry and an  $SU(3) \times SU(3)$ -structure on the generalized tangent bundle – see [17]. Thus, a supersymmetric solution of the equations of motion can then be constructed following three steps. Once one has established the existence of the  $G$ -structure, one imposes certain differential conditions taking the role of BPS-equations on it and finally studies the Bianchi identities for possible sources. Now, the authors of [16] observed that it is possible to change the differential conditions while maintaining the existence of the  $G$ -structure.<sup>1</sup> Subsequently, one obtains non-supersymmetric solutions to the full equations of motion. Supersymmetry breaking is characterized by a complex function  $\mathfrak{r} : \mathcal{M}_6 \rightarrow \mathbb{C}$ . The special case  $\mathfrak{r} = 0$  restores the differential conditions (and thus the solution) to the original SUSY ones satisfied by the geometries we mentioned previously.

As we will briefly discuss in section II, solving the DWSB solutions in full generality is still a formidable problem. Hence we will subsequently restrict our attention to Sasaki-Einstein geometries, where it is possible to restrict them further to the case of imaginary self-dual three-form flux that was already studied in [18] and the starting point of the DWSB

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<sup>1</sup> When thinking of supersymmetry in terms of spinors, this means that there is still a globally non-vanishing spinor  $\epsilon$  that does however no longer satisfy the supersymmetry equations  $\delta_\epsilon \lambda = 0$  and  $\delta_\epsilon \psi_\mu = 0$  for dilatino and gravitino.

paper. Note that [7] discusses nonsupersymmetric imaginary self-dual flux on the deformed conifold.

The outline of this paper is as follows: After introducing the DWSB equations in section II and presenting the relevant Sasaki-Einstein geometries in III, we will spend the bulk of the paper discussing the possibility of identifying the NS three-form with the  $(3, 0)$ -form  $\Omega$  that characterizes the  $SU(3)$ -structure. We find an analytic solution which breaks SUSY in the UV and restores it in the IR, yet is plagued by a naked singularity in the UV. We subsequently investigate the possibility of curing the singularity by further modifying the solution (section IV B). Here, we consider the addition of D-brane and orientifold sources following [19] and that of additional, supersymmetric flux. The latter allows us to contrast our ansatz with the Klebanov-Tseytlin [11] solution and its generalization [20]. In the appendices we summarize some conventions as well as some technical details concerning the energy-momentum tensor.

## II. THE DWSB EQUATIONS IN TYPE IIB

Let us start by summarizing a few essentials of type IIB supergravity in the democratic formalism. Except where explicitly noted, we will work in string frame. We discuss the essential ingredients here and refer more technical aspects to appendix A. In the democratic formalism of [21], the  $RR$  field strengths are doubled to  $\{F_1, F_3, F_5, F_7, F_9\}$ . The action is

$$S_s = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left\{ e^{-2\Phi} \left[ R + 4\partial\Phi^2 - \frac{1}{2}H^2 \right] - \frac{1}{4}F^2 \right\} \quad (1)$$

The sum over the different  $RR$  forms is implicit. Due to the doubling of  $RR$  fields, the Bianchi identities and  $RR$  equations of motion can be conveniently summarized in terms of polyforms as

$$d_H F = -j \quad (2)$$

Here  $j$  parametrizes possible sources. Details on the action used for the sources are relegated to appendix A 2. The twisted exterior derivative  $d_H$  is defined in equation (A3).

We are interested ten-dimensional spaces that are a warped product of the form  $\mathbb{R}^{1,3} \times_A \mathcal{M}_6$ . That is, the metric is of the form

$$ds_{10}^2 = e^{2A} dx^\mu dx_\mu + g_{ij} dy^i dy^j \quad (3)$$

while Poincaré invariance dictates the fluxes to take the form<sup>2</sup>

$$F = F + dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge e^{4A} \tilde{F} \quad \tilde{F} = \tilde{\star}_6 F \quad (4)$$

Finally we assume the existence of an  $SU(3)$ -structure which is defined in terms of an almost complex structure  $J$  and an associated  $(3, 0)$ -form  $\Omega$  that satisfy (A6).

With all conventions set up, it is time to turn to the pseudo-BPS equations of the DWSB formalism [16]. First of all there are two equations that are identical to the supersymmetric case:

$$d_H(e^{4A-\Phi} \text{Re } e^{i\theta} e^{iJ}) = e^{4A} \tilde{F} \quad (5a)$$

$$d_H(e^{2A-\Phi} \text{Im } e^{i\theta} e^{iJ}) = 0 \quad (5b)$$

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<sup>2</sup> It follows that

$$F_1 = -\star_6 \tilde{F}_5 \quad F_3 = \star_6 \tilde{F}_3 \quad F_5 = -\star_6 \tilde{F}_1$$

For supersymmetric backgrounds the set of BPS equations is completed by

$$d_H(e^{3A-\Phi}e^{-\imath\theta}\Omega) = 0 \quad (6)$$

However, in the DWSB case it is this equation that gets modified. In full generality

$$d_H(e^{3A-\Phi}e^{-\imath\theta}\Omega) = -4\imath\mathfrak{r}e^{3A-\Phi}\frac{\sqrt{\det g|_{\Pi}}}{\sqrt{\det g|_{\Pi} + R}}e^{-R}\wedge\sigma(\text{vol}_{\perp}) \quad (7)$$

Here,  $\Pi$  denotes a split of the tangent bundle  $T_*\mathcal{M}_6 = T_*\Pi \oplus T_*^{\perp}\Pi$  that also leads to a decomposition of the volume form  $\text{vol}_6 = \text{vol}_{\Pi} \wedge \text{vol}_{\perp}$ , while  $R$  is a real two-form on  $\Pi$ :  $R \in \bigwedge^2 T^*\Pi$ . Most importantly, the complex function  $\mathfrak{r}$  parametrizes the supersymmetry breaking – obviously, for  $\mathfrak{r} = 0$  one recovers the supersymmetric case. There are two further conditions given in [16]. However, since the equations are rather complicated yet in our case automatically satisfied, we relegate them to appendix B – see equations (B5) and (B6).

The Sasaki-Einstein geometries we are concerned with all satisfy  $\theta = 0$  in (5) and (7), while  $\theta = \frac{\pi}{2}$  holds for Maldacena-Núñez-like geometries. Hence we set  $\theta$  to zero and thus obtain the following equations from (5b):

$$0 = d(e^{2A-\Phi}J) \quad 0 = H \wedge J \quad (8)$$

The RR-fluxes are fully determined by (5a):

$$\begin{aligned} F_5 &= -e^{-4A} \star_6 d(e^{4A-\Phi}) \\ F_3 &= e^{-\Phi} \star_6 H \\ F_1 &= \frac{e^{-4A}}{2} \star_6 d(e^{4A-\Phi}J \wedge J) \end{aligned} \quad (9)$$

### III. SASAKI-EINSTEIN MANIFOLDS

Before continuing to study these equations, we briefly introduce two examples of Sasaki-Einstein geometry that we will use later on for explicit calculations. Our discussion follows [14] and [13]. Note that we rescale the internal part of the metric (3),

$$ds_{10}^2 = e^{2A}dx^{\mu}dx_{\mu} + e^{-2A}ds_6^2 \quad (10)$$

In general we are interested in the case where  $\mathcal{M}_6$  is a cone over a Sasaki-Einstein manifold. In other words, the metric can be written locally as

$$ds_6^2 = dr^2 + r^2 \left[ ds_4^2 + \left( \frac{d\psi'}{3} + \sigma \right) \right] \quad (11)$$

with  $ds_4^2$  locally Kähler-Einstein,  $ds_6^2$  is Calabi-Yau. There is an  $SU(3)$  structure on  $\mathcal{M}_6$  that is related to the  $SU(2)$  structure on the four-dimensional base:<sup>3</sup>

$$\Omega = e^{\imath\psi'} r^2 \Omega_4 \wedge \left[ dr + r \left( \frac{d\psi'}{3} + \sigma \right) \right] \quad J = - \left[ r^2 J_4 + r dr \wedge \left( \frac{d\psi'}{3} + \sigma \right) \right] \quad (12)$$

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<sup>3</sup> In opposite to [13], we include an additional overall minus sign in the definition of  $J$  in order to satisfy (A6).

Also, these forms satisfy a series of useful relations:

$$d(e^{\imath\psi'} r^3 \Omega_4) = 3\Omega \quad d\Omega_4 = 3\imath\sigma \wedge \Omega_4 \quad d\sigma = 2J_4$$

Specific examples are given by  $\mathbb{R}^6$ , the singular conifold  $T^{1,1}$  and the  $Y^{p,q}$  spaces of [13]. The former two turn out to be limits of the latter [13]. Their metric is given by

$$\begin{aligned}
ds_6^2 &= r^2 \left[ \sqrt{\frac{1-y}{6}} (d\theta^2 + \sin^2 \theta d\phi^2) + \frac{dy^2}{\sqrt{wq}} + \frac{\sqrt{wq}}{6} (d\beta + \cos \theta d\phi) \right. \\
&\quad \left. + \left( \frac{1}{3} d\psi' + \sigma \right)^2 \right] + dr^2 \\
\sigma &= \frac{-\cos \theta d\phi + y (d\beta + \cos \theta d\phi)}{3} \\
w(y) &= \frac{2(a-y^2)}{1-y} \quad q(y) = \frac{a-3y^2+2y^3}{a-y^2}
\end{aligned} \tag{13}$$

while the  $SU(2)$  structure is

$$\begin{aligned}
\Omega_4 &= \sqrt{\frac{1-y}{6wq}} (d\theta + \imath \sin \theta d\phi) \wedge (dy + \imath \frac{wq}{6} (d\beta + \cos \theta d\phi)) \\
J_4 &= \frac{1-y}{6} \sin \theta d\theta \wedge d\phi + \frac{1}{6} dy \wedge (d\beta + \cos \theta d\phi)
\end{aligned} \tag{14}$$

Finally, we introduce a vielbein

$$\begin{aligned}
e^1 &= r \sqrt{\frac{1-y}{6}} d\theta & e^2 &= r \sqrt{\frac{1-y}{6}} \sin \theta d\phi \\
e^3 &= \frac{r dy}{\sqrt{wq}} & e^4 &= r \frac{\sqrt{wq}}{6} (d\beta + \cos \theta d\phi) \\
e^5 &= r \left( \frac{d\psi'}{3} + \sigma \right) & e^6 &= dr
\end{aligned} \tag{15}$$

In section IV B 2 we will use the singular conifold, since the geometry is less complicated than the more general metric (13). Here, the internal metric is

$$ds_6^2 = dr^2 + r^2 \left[ \frac{1}{6} \sum_i (d\theta_i^2 + \sin^2 \theta_i d\phi_i^2) + \frac{1}{9} \left( d\psi' - \sum_i \cos \theta_i d\phi_i \right)^2 \right] \tag{16}$$

## IV. SUPERSYMMETRY-BREAKING FLUXES

### A. The ansatz

There is a difficulty one encounters immediately when solving (7). The right hand side of the equation takes the form of a complex phase multiplying a real polyform, while  $\Omega$  on the left hand side is intrinsically complex. That is, we have

$$\text{l.h.s.} \in \Omega^*(\mathcal{M}_6)^{\mathbb{C}} \quad \text{r.h.s.} \in \mathbb{C} \otimes \Omega^*(\mathcal{M}_6) \tag{17}$$

The explicit examples in [16] are often based on compactifications on toric manifolds where it is possible to perform an operation along the lines of separating the real and imaginary

parts of  $\Omega$  in a sensible way, yet for the geometries we are interested that seems not possible.<sup>4</sup> Hence, the simplest way to address the above is to set  $\text{vol}_\perp = \text{vol}_6$  and  $R = 0$ . This gives us the following equations

$$0 = d(e^{3A-\Phi}\Omega) \quad (18a)$$

$$H \wedge \Omega = 4\imath \mathfrak{r} \text{vol}_6 \quad (18b)$$

Hence it is the three-form flux  $H$  (and thus  $F_3$ ) that breaks supersymmetry, while one can expect the BPS-equations of the metric to remain unchanged. However, the additional flux will deform the solutions since it appears both in the Einstein equations as well as the relevant Bianchi identities.

All in all, the NS sector is governed by the BPS equations (5) and (18a) as well as the conditions  $H \wedge J = 0$  and  $dH = 0$  on the NS-flux. The RR fluxes can then be read off from (5a). (18b) can be solved for  $\mathfrak{r}$  and is thus not a condition. Finally one needs to impose the Bianchi identities (2) for suitable sources. Note that since we set  $\theta = 0$ , there can be no D5 sources.

Let us for the moment assume that the warp-factor depends only on  $r$ . We also take the dilaton to be constant,  $\Phi = \Phi_0$ . It follows that  $F_1 = 0$ . Concerning the NS three-form, we make the ansatz

$$H = e^{3A-\Phi_0}(\mathfrak{h}\Omega + \bar{\mathfrak{h}}\bar{\Omega}) \quad \mathfrak{h} \in \mathbb{C}, \mathfrak{h} = \text{const} \quad (19)$$

which corresponds to

$$\mathfrak{r} = -2e^{3A-\Phi_0}\bar{\mathfrak{h}} \quad (20)$$

Note that for a fairly typical warp factor this means that the scale of SUSY-breaking will grow towards the UV, with possibility of supersymmetry emerging in the IR. Since  $d(e^{\imath\psi'}r^3\Omega_4) = 3e^{3A-\Phi_0}\Omega$ , it follows that the corresponding two-form potential is given by

$$B = \frac{r^3}{3}e^{-\Phi_0}(\mathfrak{h}e^{\imath\psi'}\Omega_4 + \bar{\mathfrak{h}}e^{-\imath\psi'}\bar{\Omega}_4) \quad (21)$$

One can also immediately calculate  $F_3 = e^{-\Phi} \star_6 H$ .

$$F_3 = -\imath e^{3A-2\Phi_0}(\mathfrak{h}\Omega - \bar{\mathfrak{h}}\bar{\Omega}) \quad (22)$$

The RR three-form satisfies the Bianchi identity, which in the absence of  $F_1$  reduces to closure of the form:  $dF_3 = 0$ . The final step, is to impose

$$dF_5 + H \wedge F_3 = 0 \quad (23)$$

Here we have

$$\begin{aligned} H \wedge F_3 &= -16e^{6A-3\Phi_0}\mathfrak{h}\bar{\mathfrak{h}}\text{vol}_6 \\ dF_5 &= \frac{4}{r}e^{2A-\Phi_0}[rA'' - 4r(A')^2 + 5A']\text{vol}_6 \end{aligned} \quad (24)$$

The resulting ODE can be solved analytically. Writing the result in terms of the more conventional warp-factor  $A = -\frac{1}{4}\log h$ :

$$h = h_0 + \frac{h_1}{r^4} - \frac{4}{3}e^{-2\Phi_0}r^2\mathfrak{h}\bar{\mathfrak{h}} \quad (25)$$

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<sup>4</sup> One can see this quite explicitly by studying the  $SU(3)$ -structure defined in (12).

As we anticipated from (20), the modifications appear only in the UV. However, they are highly problematic. At sufficiently large  $r$   $h(r)$  becomes negative. Since the Ricci scalar behaves as  $R \sim h^{-3/2}$ , there is a naked singularity.

To understand things a bit better, it is appropriate to take a look at the Einstein equation and the energy-momentum tensor explicitly. Instead of doing so straightaway however, let us generalize the above discussion, by contrasting the above ansatz for  $H$  with supersymmetric flux. In the case of the conifold this is provided by the Klebanov-Tseytlin background, which can be generalized to encompass the  $Y^{p,q}$  geometries that we discussed in section III. This was done in [20]. (One can actually obtain Klebanov-Tseytlin solution as a limit of [20].) So in the following we work explicitly with (13).

The equations for  $H$  are linear, allowing us to use both the supersymmetry preserving three-form as well as the SUSY-breaking choice

$$H = H_{\text{susy}} + H_{\text{nosy}}$$

$$H_{\text{susy}} = (k_1 d + k_2 \star_6 d) \left[ \frac{3e^{2A} \log r}{2\pi r^2 (1-y)^2} (e^3 \wedge e^4 - e^1 \wedge e^2) \right] \quad (26)$$

and  $H_{\text{nosy}}$  as above. Since  $H_{\text{susy}} \wedge \Omega = 0$ ,  $\mathfrak{r}$  remains unchanged. As we will see, the constants  $k_1$  and  $k_2$  enter the background metric only in the form of  $k_1^2 + k_2^2$ .  $H_{\text{susy}}$  contributes to  $F_3$  as

$$F_{\text{susy}} = e^{-\Phi_0} (k_1 \star_6 d - k_2 d) \left[ \frac{3e^{2A} \log r}{2\pi r^2 (1-y)^2} (e^3 \wedge e^4 - e^1 \wedge e^2) \right] \quad (27)$$

$$= \frac{3e^{3A-\Phi_0}}{2\pi r^3 (1-y)^2} (e^1 \wedge e^2 - e^3 \wedge e^4) \wedge (e^5 k_1 + e^6 k_2)$$

Writing the warp factor once again in terms of  $h(r, y)$ , the Bianchi identity for  $F_5$  reduces to the following PDE

$$0 = r \partial_r [2e^{2\Phi_0} \pi^2 r^5 (1-y)^4 \partial_r h] + \partial_y [2e^{2\Phi_0} \pi^2 r^4 (1-y)^4 q w \partial_y h] \quad (28)$$

$$+ 6e^{2\Phi_0} \pi^2 r^4 (1-y)^3 q w \partial_y h + 32\pi^2 r^6 (1-y)^4 \mathfrak{h} \bar{\mathfrak{h}} + 9e^{2\Phi_0} (k_1^2 + k_2^2)$$

Following [20], we make the ansatz  $h(r, y) = r^{-4} [h_r(r) + h_y(y)]$ . This leads to the ODE for  $h_r$

$$0 = 16r^5 \mathfrak{h} \bar{\mathfrak{h}} + e^{2\Phi_0} (r h_r'' - 3h_r') \quad (29)$$

which is solved by

$$h_r = h_0 r^4 + h_1 + h_2 \log r - \frac{4}{3} e^{-2\Phi_0} \mathfrak{h} \bar{\mathfrak{h}} r^6 \quad (30)$$

which is identical to (25). Subsequently, one is left with another ODE, this time in terms of  $y$ :

$$\partial_y [2\pi^2 (1-y) q w \partial_y h_y] = \frac{8h_2 \pi^2 (1-y)^4 - 9(k_1^2 + k_2^2)}{(1-y)^3} \quad (31)$$

This can be integrated. Since  $\partial_y h_y$  is finite at  $y_i$  and  $q(y_i) = 0$ , the integral of the right hand side has to vanish. This constraint yields

$$h_2 = \frac{9(k_1^2 + k_2^2)}{8\pi^2 (1-y_2)^2 (1-y_1)^2} \quad (32)$$

fixing  $h_2$ . The equation for  $y$  can then be solved as in [20]. We are mostly interested in the radial behavior and note that the contribution of the supersymmetric part of the flux does not allow us to cure the singularity. It is a curious fact that the different parts of the flux separate so nicely in the PDE for  $h(r, y)$ . As soon as we split the equation into two ODEs, the  $H_{\text{susy}}$  is only relevant for  $h_y$ , while  $H_{\text{nosy}}$  appears in the equation for  $h_r$ .

Having introduced the supersymmetric three-forms, let us take a look at the Einstein equation and the energy-momentum in Einstein frame, including possible sources. The technical aspects of the discussion are of course very similar to [18]. From (A14) we find

$$0 = R_{\mu\nu} + \frac{1}{8}g_{\mu\nu}(e^{-\Phi}H^2 + e^{\Phi}F_3^2) - \frac{1}{4}F_\mu^5 \cdot F_\nu^5 + 2\kappa_{10}^2 \left[ T_{\mu\nu}^{\text{src}} - \frac{1}{8}g_{\mu\nu}(T^{\text{src}})^m_m \right] \quad (33)$$

So, using (A16), this becomes<sup>5</sup>

$$e^{4A}\eta_{\mu\nu}\hat{\nabla}^2 A = \frac{1}{4}e^{2A-\frac{3\Phi_0}{2}}\eta_{\mu\nu}H^2 + 4\eta_{\mu\nu}e^{4A}g^{ij}\partial_i A\partial_j A + 2\kappa_{10}^2 T_p \frac{7-p}{16}g_{\mu\nu} \frac{\sqrt{-g_{p+1}}}{\sqrt{-g}}\delta(\Sigma) \quad (34)$$

Substituting

$$H^2 = \frac{e^{6A-\frac{\Phi}{2}}}{2} \left[ 9e^{2\Phi} \frac{k_1^2 + k_2^2}{\pi^2 r^6 (1-y)^4} + 32\mathfrak{h}\bar{\mathfrak{h}} \right] \quad (35)$$

we find<sup>6</sup>

$$e^{4A}\eta_{\mu\nu}\hat{\nabla}^2 A = \eta_{\mu\nu} \left\{ \frac{1}{8}e^{8A-2\Phi_0} \left[ 9e^{2\Phi_0} \frac{k_1^2 + k_2^2}{\pi^2 r^6 (1-y)^4} + 32\mathfrak{h}\bar{\mathfrak{h}} \right] + 4e^{4A}g^{ij}\partial_i A\partial_j A \right\} + 2\kappa_{10}^2 T_p \frac{7-p}{16}g_{\mu\nu} \frac{\sqrt{-g_{p+1}}}{\sqrt{-g}}\delta(\Sigma) \quad (36)$$

As one would expect, this is the same PDE (28) that we previously obtained from the Bianchi identity. Here one sees very nicely the difference between the SUSY and the non-SUSY fluxes. The former disappear in the UV and dominate the IR, while the latter become only relevant in the UV. Finally, note that the source-term for space-time filling  $D_3$  sources is

$$\frac{2\kappa_{10}^2 T_3}{4}g_{\mu\nu} \frac{\sqrt{-g_{p+1}}}{\sqrt{-g}}\delta^6(y^i) = \frac{2\kappa_{10}^2 T_3}{4}\eta_{\mu\nu}e^{8A+\Phi_0} \frac{\delta^6(y^i)}{\sqrt{\hat{g}}} \quad (37)$$

Hence – as long as the dilaton is constant – the three-form fluxes contribute to the energy momentum tensor as a continuous distribution of D3 branes would.

## B. Instead of a no-go theorem

One can think of several ways to improve the situation and deal with the singularity. As we will see – and as one can check by performing explicit calculations – none of these

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<sup>5</sup> Note that the spatial components of the five-form are given by

$$F_5 = d(e^{4A-\Phi}) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$$

which gives (in Einstein frame)

$$F_\mu^5 \cdot F_\nu^5 = -\eta_{\mu\nu}e^{4A}g^{ij}\partial_i(4A-\Phi)\partial_j(4A-\Phi)$$

<sup>6</sup> Since we are not concerned with compactifications, we cannot use the argument of [18] that the right hand side has to vanish.



succeed. Thus, it is tempting to think that there might be a no-go theorem<sup>7</sup> that does not allow for imaginary self-dual SUSY-breaking three-form flux on Sasaki-Einstein spaces in the context of supergravity.

There are several simple methods one can think of to change the PDE (28) that we obtained from the Bianchi identity. The first of these is changing the relative sign between  $F_3$  and  $H$ . This Effectively changes the sign in (30), leading to a warp factor that (ignoring  $y$ ) behaves as  $r^{-4}$  in the IR and  $r^2$  in the UV, again with supersymmetry breaking in the UV. However, flipping the sign between the two three-forms is inconsistent. First of all, the Einstein equation is invariant under this transformation. Yet as we saw, (36) is equivalent to (28), and changing the sign would only modify the latter. Furthermore there is the issue of the equation of motion for  $F_3$  – i.e. the Bianchi identity for  $F_7$ . The sign here changes as well leading to a further inconsistency.<sup>8</sup>

### 1. Additional source terms

Sticking with more physical possibilities, one can look into modifying the Bianchi identity and Einstein equation by the addition of backreacting sources. The simplest of course are space-time filling objects with codimension 6. That is 3-branes and O3-planes. From the Bianchi identity (2) it follows that the appropriate source term is  $j_6$ . One has to be careful with the sign here. As discussed in the appendix A 2, 3-branes couple to the four-form potential via  $T_3 \int C_4 \wedge \sigma(j_6)$ , yet  $\sigma(j_6) = -j_6$ . We can write a generic source as

$$j_6 = -e^{-3\Phi_0} \rho \text{vol}_6 \quad (38)$$

for a generic density function  $\rho : \mathcal{M}_6 \rightarrow \mathbb{R}$ . Once can deal with the  $y$ -dependence as in section IV A, so for simplicity we assume that  $h$  depends on  $r$  alone. The resulting ODE is

$$0 = r(16\mathfrak{h}\bar{\mathfrak{h}} + \rho) + e^{2\Phi_0}(5h' + rh'') \quad (39)$$

It becomes clear that only a negative, constant  $\rho$  allows to cancel the three-form flux charge. This corresponds to a constant background charge density extending all the way to the far UV, a rather problematic assumption. Since this analysis concerns only the Bianchi identity, the question remains whether the additional charge is that of anti D3-branes or of orientifold planes. Here, the Einstein equation (36) confirms that we are actually dealing with negative tension objects.

For completeness, let us look into possible radial dependence of  $\rho$ . In this case the new branes are smeared over the Sasaki-Einstein base. The simplest example  $\rho(r) = \delta(r - r_1)$  changes the warp factor by a further negative term:

$$h(r) \mapsto h(r) - e^{-2\Phi_0} \frac{r_1}{4} \left(1 - \frac{r_1^4}{r^4}\right) \theta(r - r_1) \quad (40)$$

<sup>7</sup> Of course, one has to be slightly careful here since nonsupersymmetric deformations have been found that do not encounter the issue of a UV singularity – see e.g. [7].

<sup>8</sup> Since the change of sign leads to an interesting PDE, one can try to change the sign in both the Bianchi identity and the Einstein equation. However, the contribution to the latter is of course positive definite. Hence, there is only the formal possibility of making the flux imaginary. The substitution  $H \mapsto \imath H$ ,  $F_3 \mapsto \imath F_3$  gives indeed a solution to the equations of motion. As a matter of fact, introducing a phase factor  $H \mapsto e^{\imath\eta} H$ , one ends up with a warp factor

$$h(r) = h_0 + \frac{h_1}{r^4} - \frac{4}{3} e^{2\imath\eta - 2\Phi_0} r^2 \mathfrak{h}\bar{\mathfrak{h}}$$

In light of the fact that the relevant PDEs are all linear in  $h$ , this result should not be surprising.

Before we move on to D7-branes, let us briefly consider another possibility: Since we have  $h \rightarrow 0$  at the singularity, it might be possible to reconsider the solution as a compact one. I.e. for  $h$  as in (25) with  $h_0 = 0$  the warp-factor vanishes at  $r_0 = \left(\frac{3h_1}{4\mathfrak{h}}\right)^{1/6}$  and thus the volume of the internal manifold goes to zero. If one considers the geometry as defined on the compact interval  $r \in [0, r_0]$ , with a suitable negative tension source – that we found in the last paragraph – at  $r_0$  to cancel tadpoles. Yet by analyzing the metric around  $r_0$ , we find that the singularity is not conical.<sup>9</sup>

## 2. D7 sources on the singular conifold

So finally we turn to the inclusion of D7-branes, which corresponds to a non-constant dilaton in the background. The general idea is that the orientation of the branes and the three-form flux is such that the  $H$ -field induces negative D3-charge onto the branes, canceling or at least softening the effect of the SUSY-breaking terms in the Bianchi identity for the five-form. Furthermore the inclusion of such branes will induce a non-trivial profile for the dilaton, which might also help matters. The inclusion of backreacting D7-branes in Sasaki-Einstein geometries was first studied in [19]. See also [22] and [23]. In general the D7-branes are embedded in such a way that the  $U(1)$ -fiber in the five-dimensional Sasaki-Einstein space is deformed. I.e. we need to make an ansatz for the metric (16) allowing for such deformations and introduce here the relevant vielbein:

$$\begin{aligned} e^1 &= \frac{e^g}{\sqrt{6}} d\theta_1 & e^2 &= \frac{e^g}{\sqrt{6}} \sin \theta_1 d\phi_1 \\ e^3 &= \frac{e^g}{\sqrt{6}} d\theta_2 & e^4 &= \frac{e^g}{\sqrt{6}} \sin \theta_2 d\phi_2 \\ e^5 &= e^f dr & e^6 &= \frac{e^f}{3} (d\psi' - \sum_i \cos \theta_i d\phi_i) \end{aligned} \tag{41}$$

Note that in opposite to the above references we also include the warp factor for the internal space –  $e^{-2A}$  – in the ten-dimensional geometry and choose a different radial coordinate. The  $SU(3)$  structure is given by

$$\begin{aligned} J &= e^1 \wedge e^2 + e^3 \wedge e^4 + e^5 \wedge e^6 \\ \Omega &= e^{\psi'} (e^1 + \imath e^2) \wedge (e^3 + \imath e^4) \wedge (e^5 + \imath e^6) \end{aligned} \tag{42}$$

While the SUSY-breaking flux remains as in (19) (except that the dilaton is no longer constant), we generalize the ansatz for the SUSY flux slightly to allow for different radial

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<sup>9</sup> To understand this in principle, let us take the warp factor to be of the form  $h = r^{-4} - r^2$ . If one introduces a new coordinate via  $r = 1 - \frac{\rho^4}{6}$ , one finds

$$\sqrt{h}(dr^2 + r^2 ds_5^2) = \rho^2 ds_5^2 - \frac{5}{24} \rho^6 ds_5^2 + \frac{4}{9} \rho^8 d\rho^2 + \mathcal{O}(\rho^9)$$

That is, it is not possible to locally write the metric as  $d\rho^2 + \rho^2 ds_5^2$  and one can only satisfy the relevant junction conditions by assuming that there is a *thick* object with negative tension at  $r_0$ ; again, not a physically viable assumption.

dependence.

$$B_{\text{susy}} = k_1(r)(\sin \theta_1 d\theta_1 \wedge d\phi_1 - \sin \theta_2 d\theta_2 \wedge d\phi_2) + \frac{k_2}{3}(\cos \theta_1 \cos \theta_2 d\phi_1 \wedge d\phi_2 - \cos \theta_1 d\phi_1 \wedge d\psi + \cos \theta_2 d\phi_2 \wedge d\psi) \quad (43)$$

Proceeding as usual, one finds BPS-like-equations

$$f' = 3 - 2e^{2f-2g} \quad g' = e^{2f-2g} + \frac{\Phi'}{2} \quad (44)$$

When it comes to imposing the Bianchi identities, things are a bit more complicated due to the D7s. Also, instead of using the forms  $j_i$  appearing in (2) it is actually simpler to use the  $\Theta_i$  defined in (A12). The reason is that the  $j_i$  do not just correspond to sources alone, but also to induced charge, as one can see from the form of the Wess-Zumino term when written in terms of  $j$  – see (A8).

Again, there are no five-brane source terms and therefore we need to impose  $\Theta_4 = 0$ . We have

$$\begin{aligned} \Theta_4 = & 2\sqrt{6}e^{4A-f-3g-\Phi}k_2\Phi'(\cot \theta_2 e^{1246} - \cot \theta_1 e^{2346}) \\ & + \frac{2}{3}e^{4A-f-2\Phi}[\text{Re}(\mathfrak{h}e^{\psi'})e^{1356} + \text{Im}(\mathfrak{h}e^{\psi'})(e^{1456} + e^{2356})][(\Phi')^2 - \Phi''] \\ & + \frac{2}{3}e^{4A-2(f+g+\Phi)}[3e^\Phi k_2 \cot \theta_1 \cot \theta_2 + e^{f+2g}\text{Re}(\mathfrak{h}e^{\psi'})]e^{2456}(\Phi')^2 - \Phi'' \\ & + 6e^{4A-2(f+g)-\Phi}(e^{1256} - e^{3456})\{2k_1'\Phi' - k_1'' + k_1[\Phi'' - (\Phi')^2]\} \end{aligned} \quad (45)$$

For this to vanish, we have to impose

$$\Phi = \Phi_0 - \log(r + r_0) \quad k_1 = -\frac{\tilde{k}_1}{r + r_0} \quad k_2 = 0 \quad (46)$$

In contrast to the previous case, there is no longer equivalence between  $H$  and its Hodge dual, which is of course due to the fact now the Bianchi identity for  $F_3$  is twisted by  $F_1$ . An interesting difference to the supersymmetric case is that we are not free to choose the dilaton. One can immediately calculate

$$\begin{aligned} F_1 &= \frac{e^{-\Phi_0}}{3}(d\psi' - \cos \theta_1 d\phi_1 - \cos \theta_2 d\phi_2) \\ \Theta_2 &= \frac{1}{3}e^{-\Phi_0}(\sin \theta_1 d\theta_1 \wedge d\phi_1 + \sin \theta_2 d\theta_2 \wedge d\phi_2) \end{aligned} \quad (47)$$

It follows that the additional D7 sources correspond to massless flavors in the gauge theory. This is in stark contrast to the supersymmetric case, where there are solutions dual to massive flavors – see [22]. However, one can see directly from the form of  $\Theta_4$  that setting  $\mathfrak{h} \rightarrow 0$  removes any condition on the dilaton.

Since the dilaton is no longer arbitrary, the BPS-like equations for  $f$  and  $g$  can be solved.

$$\begin{aligned} f &= e^{2c_f}3 + 3r + r_0 - \frac{1}{3}\log\left[54c_g + e^{6r}\left(r + r_0 - \frac{1}{6}\right)\right] \\ g &= \frac{1}{6}\left\{4c_g + 6r_0 - 3\log(r + r_0) + \log\left[54c_g + e^{6r}\left(r + r_0 - \frac{1}{6}\right)\right]\right\} \end{aligned} \quad (48)$$

Finally, we have to impose  $\Theta_6 = 0$ . This gives the following ODE:

$$0 = 4e^{4A}[2e^{2f+4g}(r+r_0)^6\mathfrak{h}\bar{\mathfrak{h}} + 9e^{2\Phi_0}\tilde{k}_1] + e^{2(g+\Phi_0)}(r+r_0)^2\{-2e^{2f}(r+r_0)[1+4A'(r+r_0)] + e^{2g}[1+2(r+r_0)(2A'+4(r+r_0)(A')^2-(r+r_0)A'')]\} \quad (49)$$

In order to discuss (49), let us impose  $c_g = c_f = 0$  and  $r_0 = \frac{1}{6}$ .  $c_f$  and  $c_g$  are simply scales for the two internal two-spheres while  $r_0$  can be expected to set the minimum for the radial coordinate. Since we are mainly interested in the question whether it might be possible to cure the UV singularity, this should be a good assumption. Similarly, setting the supersymmetric part of the three-form flux to zero is also reasonable. With these assumptions, let us first discuss the SUSY solutions to (49) – i.e. we set  $\mathfrak{h} \rightarrow 0$ . Then working in terms of the warp factor  $h(r)$ , we find

$$h(r) = (6r+1)(h_0 + h_1\Gamma_{\frac{1}{3}}(4r)) + \frac{36 \times 2^{1/3}r^{2/3}\tilde{k}_1^2[3 \times 2^{2/3}r^{1/3} - 2e^{4r}(6r+1)\Gamma_{\frac{1}{3}}(4r)]}{(e^{6r+1}r)^{2/3}} \quad (50)$$

with the incomplete gamma function

$$\Gamma_a(z) = \int_z^\infty t^{a-1}e^{-t}dt \quad (51)$$

One can check explicitly that this leads to a well-defined metric. That is, as long as  $\tilde{k}_1$  is not too large, we have  $h > 0$ .

Turning to the non-SUSY case, we take set  $\tilde{k}_1$  to zero as well to keep things manageable. That should be reasonable since we have already seen previously that the additional SUSY flux does not cure the singularity. One finds

$$h = \frac{6r+1}{18} \left\{ 18h_0 + 9 \times 2^{1/3}h_1\Gamma_{\frac{1}{3}}(4r) + 2^{5/3}(-1)^{1/3}\mathfrak{h}\bar{\mathfrak{h}}\Gamma_{\frac{4}{3}}(-2r) \right\} \quad (52)$$

Here, both the factor  $(-1)^{1/3}$  and the gamma function with negative argument are complex, showing that it is not possible to add D7 branes while breaking supersymmetry.

### 3. A different choice for the three-form flux

Let us finally turn to what is probably the most obvious question – whether there is a different three-form that one can use for the SUSY-breaking flux instead of the obvious choice we used so far, equation (19).

The problem is that the flux has not only to satisfy the algebraic relation  $H \wedge J = 0$ , but has also to satisfy the various Bianchi identities. Not least of all it has to be well-defined. Let us give two examples.

In the case of the conifold, there is

$$\frac{e^{f+2g}}{6} \sin \theta_1 \sin \theta_2 dr \wedge d\theta_1 \wedge d\theta_2 \quad (53)$$

yet here it is not possible to impose the Bianchi identity for  $F_3$ . In the case of  $\text{AdS}_5 \times S^5$  there are several possible ansätze, since the internal geometry is  $\mathbb{R}^6$ . As an example, let us choose

$$B = b(y_1)dy_3 \wedge dy_5 \quad (54)$$

where we have introduced cartesian coordinates  $y_i$  on  $\mathbb{R}^6$ . The Bianchi identity for  $F_3$  leads to the linear relation

$$b(y_1) = b_1 y_1 \quad (55)$$

while we find for the warp-factor ( $y^2 = \sum_i y_i^2$ )

$$A(y) = -\frac{1}{4} \log \left( h_0 + \frac{h_0}{y^4} - \frac{b_1^2}{12} y^2 \right) \quad (56)$$

Which has the same UV singularity as (25). This is a reasonable result, since in both cases we add constant flux to the background geometry.

In general, if the form  $J$  is given by

$$J = \sum_{i=1}^3 e^i \wedge e^{i+3} \quad (57)$$

the algebraic relation is satisfied by the eight three-forms

$$e^i \wedge e^j \wedge e^k \quad i \in \{1, 4\}, j \in \{2, 5\}, k \in \{3, 6\} \quad (58)$$

which are in general often ill-defined though and certainly not necessarily closed. This is not the only class of three-forms satisfying the relation as can be understood from the Klebanov-Tseytlin case. Here, one makes use of the fact that there are two non-trivial two-cycles in the Kähler-Einstein base (see [13]). The ansatz for the flux is then based on a linear combination of the two and satisfies  $H \wedge J = 0$  because the contribution from each form cancels that of the other. This is straightforward to see in the case of the conifold – see the first line in our ansatz for the supersymmetric flux there, equation (43). If one tries to make this sort of ansatz for the SUSY-breaking case, one always finds  $H \wedge \Omega = 0$ .

## V. CONCLUSIONS

In the main part of this paper, we attempted a rather exhaustive study of nonsupersymmetric imaginary self-dual three-form flux on Sasaki-Einstein backgrounds. While we were able to find analytic solutions, all of these were plagued by problematic issues, most notably a naked curvature singularity in the UV that raises the question whether there is a well-defined UV completion. Since all our attempts at curing the singularity failed one might conjecture that it is not possible to do so within the context of supergravity. Comparisons with the supersymmetric flux suggest that this is due to the fact that the flux does not disappear in the UV, yet the radial behavior of the flux is fully determined by the Bianchi identities on  $H$  and  $F_3$ .

So far, we have tried to remove the singularity by modifying the differential equations resulting from the Bianchi identity and Einstein equations – without success. An alternative is to try to push the singularity to infinity by rescaling the solution in a suitable fashion. From a first study it appears, that when doing so one usually ends up with a supersymmetric solution without three-form flux, yet this question is not fully answered.

Of course, not all singularities in supergravity backgrounds are problematic in the context of string theory, and therefore it might be possible to find a more suitable UV completion for the backgrounds discussed. Yet in the context of gauge/string duality it would be certainly

more desirable to study solutions that restore SUSY in the UV and break it in the IR instead of the opposite.

One has to wonder whether it might be possible to improve the situation by finding a way to modulate the behavior of the flux for large  $r$ . While this does not seem possible within the DWSB framework, one might be able to combine the ansätze presented here with the alternative approaches mentioned in the introduction. E.g. even maintaining the ansatz (19) with  $\Phi \neq \Phi_0$  one should obtain a limited amount of total flux as long as the dilaton grows sufficiently fast towards the UV.

Let us finish with the remark that both the DWSB approach as well as the further restrictions we have imposed here limit the possible solutions considerably. Hence to make progress with the idea of maintaining the existence of a  $G$ -structure while breaking supersymmetry, it should be instructive to study the  $G$ -structure and the violation of the associated BPS-equations in the case of known non-supersymmetric deformations such as [5] and [7].

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## Appendix A: Conventions

### 1. Differential geometry

We use the same conventions as in [16] and [24]. The ten- and six-dimensional Hodge duals are defined as follows

$$\begin{aligned}\star_6\omega &= \frac{\sqrt{g_6}}{p!(6-p)!}\epsilon_{m_1\dots m_6}\omega^{m_7-p\dots m_6}dy^{m_1}\wedge\dots\wedge dy^{m_{6-p}} \\ \star_{10}\omega &= -\frac{\sqrt{-g_{10}}}{p!(10-p)!}\epsilon_{M_1\dots M_{10}}\epsilon_{M_1\dots M_{10}}\omega^{M_{11-p}\dots M_{10}}dx^{M_1}\wedge\dots\wedge dx^{M_{10-p}}\end{aligned}\tag{A1}$$

As a consequence, we have

$$\eta\wedge\star_6\lambda = (-1)^{p(6-p)}\frac{1}{p!}\eta_{m_1\dots m_p}\lambda^{m_1\dots m_p}\text{vol}_6\tag{A2}$$

$d_H$  is the twisted exterior derivative and acts on forms as

$$d_H\omega = (d + H\wedge)\omega\tag{A3}$$

There is also a modified hodge dual  $\tilde{\star}_6$ , that includes the action of the operator  $\sigma$ :

$$\tilde{\star}_6 \equiv \star_6 \circ \sigma\tag{A4}$$

where  $\sigma$  reverses all indices of a  $p$ -form

$$\sigma(\omega) = \frac{1}{p!} \omega_{M_p \dots M_1} dx^{M_1} \wedge \dots \wedge dx^{M_p} = (-1)^{\frac{(p-1)p}{2}} \omega \quad (\text{A5})$$

The forms  $J$  and  $\Omega$  defining the  $SU(3)$  structure satisfy the following relations

$$\frac{1}{3!} J \wedge J \wedge J = -\frac{i}{8} \Omega \wedge \bar{\Omega} = \text{vol}_6 \quad \text{and} \quad \star_6 \Omega = -i \Omega \quad (\text{A6})$$

For an  $n$ -dimensional internal space (in our case,  $n = 6$ ), the Mukhai pairing and its generalization are defined as follows

$$\langle \omega, \chi \rangle = \omega \wedge \sigma(\chi)|_n \quad \langle \omega, \chi \rangle_k = \omega \wedge \sigma(\chi)|_k \quad (\text{A7})$$

## 2. Type IIB supergravity

The source-terms  $j$  arise from the action of space-time filling branes, described by

$$\begin{aligned} S_{\text{src}} &= -T_p \int_{\mathcal{M}_{10}} \Psi \wedge \sigma(j) + T_p \int_{\mathcal{M}_{10}} \frac{C + \tilde{C}}{2} \wedge \sigma(j) \\ &= -T_p \int_{\Sigma} e^{-\Phi} \sqrt{-g_{\text{ind}} + \mathcal{F}} + T_p \int_{\Sigma} \frac{C + \tilde{C}}{2} \wedge e^{\mathcal{F}} \end{aligned} \quad (\text{A8})$$

where  $\Sigma$  is the branes' world volume. The calibration form  $\Psi$  is<sup>10</sup>

$$\Psi = e^{-\Phi} \text{vol}_{1,3} \wedge \text{Re}(e^{i\theta} e^{iJ}) \quad (\text{A9})$$

The  $\tilde{C}$  appearing in (A8) are the dual potentials. I.e. the RR field strengths are related to their potentials via

$$F = d_H C = \star_{10} d_H \tilde{C} \quad (\text{A10})$$

There is a subtlety when matching the components of the polyform  $j$  with possible sources, since  $j$  also accounts for induced charges. In other words, the presence of say a D7-brane can induce a D3 charge, leading to  $j_6 \neq 0$  in the absence of D3 sources. To account for this, one can introduce a further polyform  $\Theta$  and write the Wess-Zumino term as

$$T_p \int_{\mathcal{M}_{10}} \frac{C + \tilde{C}}{2} \wedge \sigma(j) = T_p \int_{\mathcal{M}_{10}} \frac{C + \tilde{C}}{2} \wedge e^{\mathcal{F}} \wedge \Theta \quad (\text{A11})$$

Matching terms on both sides, one finds

$$\begin{aligned} j_2 &= -\Theta_2 \\ j_4 &= \Theta_4 + B \wedge \Theta_2 \\ j_6 &= -(\Theta_6 + B \wedge \Theta_4 + \frac{1}{2} B^2 \wedge \Theta_2) \\ j_8 &= \Theta_8 + B \wedge \Theta_6 + \frac{1}{2} B^2 \wedge \Theta_4 + \frac{1}{3!} B^3 \wedge \Theta_2 \end{aligned} \quad (\text{A12})$$

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<sup>10</sup> With

$$\text{vol}_{1,3} = e^{4A} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$$

We will use the Einstein frame<sup>11</sup> only when studying the energy-momentum tensor. Thus the use of the democratic formalism is rather cumbersome. Hence we drop  $F_7$  and  $F_9$  when working in Einstein frame:

$$S_E = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[ R - \frac{1}{2} \partial \Phi^2 - \frac{e^{-\Phi}}{2} H^2 - \frac{1}{2} \left( e^{2\Phi} F_1^2 + e^\Phi F_3^2 + \frac{1}{2} F_5^2 \right) \right] \quad (\text{A13})$$

This leads to an equation of motion

$$\begin{aligned} 0 &= R_{mn} - \frac{1}{2} \partial_m \Phi \partial_n \Phi - e^{-\Phi} 2H_m H_n + \frac{1}{8} g_{mn} (e^{-\Phi} H^2 + e^\Phi F_3^2) \\ &\quad - \frac{1}{2} \left( e^{2\Phi} F_m^1 F_n^1 + e^\Phi F_m^3 \cdot F_n^3 + \frac{1}{2} F_m^5 \cdot F_n^5 \right) + 2\kappa_{10}^2 \left( T_{mn}^{\text{src}} - \frac{1}{8} g_{mn} g^{kl} T_{kl}^{\text{src}} \right) \\ T_{mn}^{\text{src}} &= \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{mn}} S_{\text{src}} \end{aligned} \quad (\text{A14})$$

Furthermore, the Ricci tensor and scalar of the Einstein-frame metric

$$ds^2 = e^{2A - \frac{\Phi}{2}} dx^\mu dx_\mu + e^{-2A - \frac{\Phi}{2}} \hat{g}_{ij} dy^i dy^j \quad (\text{A15})$$

are given by

$$\begin{aligned} R_{\mu\nu} &= e^{4A} \eta_{\mu\nu} \left[ -\frac{1}{2} \hat{\nabla}^2 \left( 2A - \frac{\Phi}{2} \right) + \partial^i \Phi \partial_i \left( 2A - \frac{\Phi}{2} \right) \right] \\ &= -\frac{1}{2} e^{4A} \eta_{\mu\nu} \left[ e^{-2A + \frac{\Phi}{2}} \hat{\nabla}^2 e^{2A - \frac{\Phi}{2}} + \partial \left( 2A + \frac{\Phi}{2} \right)^2 - \partial \Phi^2 \right] \\ R_{ij} &= \hat{R}_{ij} + 2\hat{\nabla}_i \partial_j \Phi - 8\partial_i A \partial_j A + 2(\partial_i A \partial_j \Phi + \partial_i \Phi \partial_j A) + \frac{1}{2} \partial_i \Phi \partial_j \Phi \\ &\quad + \hat{g}_{ij} \left[ \frac{1}{2} \hat{\nabla}^2 \left( 2A + \frac{\Phi}{2} \right) - \partial^l \Phi \partial_l \left( 2A + \frac{\Phi}{2} \right) \right] \\ R &= e^{2A + \frac{\Phi}{2}} \left( \hat{R} + 2\hat{\nabla}^2 A - 8\partial A^2 + \frac{9}{2} \hat{\nabla}^2 \Phi - \frac{9}{2} \partial \Phi^2 \right) \end{aligned} \quad (\text{A16})$$

with  $R_{\mu i} = R_{i\mu} = 0$ . The  $(\mu, \nu)$  components of the Einstein tensor are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = e^{4A} \eta_{\mu\nu} \left[ -2\hat{\nabla}^2 (A + \Phi) + 4\partial A^2 + \frac{7}{4} \partial \Phi^2 + 2\partial^i \Phi \partial_i A - \frac{1}{2} \hat{R} \right] \quad (\text{A17})$$

We used that

$$\begin{aligned} \Gamma_{\mu\nu}^i &= -\frac{1}{2} e^{4A} \eta_{\mu\nu} \hat{\partial}^j \left( 2A - \frac{\Phi}{2} \right) & \Gamma_{\mu i}^\lambda &= \frac{1}{2} \delta_\mu^\lambda \partial_i \left( 2A - \frac{\Phi}{2} \right) \\ \Gamma_{ij}^l &= \hat{\Gamma}_{ij}^l + \frac{1}{2} \left[ (\delta_j^l \partial_i + \delta_i^l \partial_j - \hat{g}_{ij} \hat{\partial}^l) \left( -2A - \frac{\Phi}{2} \right) \right] \end{aligned} \quad (\text{A18})$$

<sup>11</sup> Recall that  $(g_s)_{MN} = e^{\frac{\Phi}{2}} (g_E)_{MN}$ . Furthermore, under Weyl transformations of the metric, the Ricci scalar behaves as

$$R[e^{\alpha\Phi} g] = e^{-\alpha\Phi} \left\{ R[g] - (D-1)\alpha \nabla^2 \Phi - \frac{(D-1)(D-2)}{4} \alpha^2 \partial \Phi^2 \right\}$$



## Appendix B: The full set of DWSB equations

We list the DWSB equations that were derived in [16] in terms of the pure spinors  $\Psi_{1,2}$ . In the special case of type IIB  $SU(3)$ -structure backgrounds, these take the form

$$\Psi_1 = e^{\imath\theta} e^{\imath J} \quad \Psi_2 = e^{-\imath\theta} \Omega \quad (\text{B1})$$

We begin with those equations that remain the same as in the supersymmetric case

$$d_H(e^{4A-\Phi} \text{Re } \Psi_1) = e^{4A} \tilde{F} \quad d_H(e^{2A-\Phi} \text{Im } \Psi_1) = 0 \quad (\text{B2})$$

The equation for  $\Psi_2$  however is modified from its supersymmetric form to

$$d_H(e^{3A-\Phi} \Psi_2) = 4\imath \mathfrak{r} (-1)^{|\Psi_2|} e^{3A-\Phi} \frac{\sqrt{\det g|_{\Pi}}}{\sqrt{\det(g|_{\Pi} + R)}} e^{-R} \wedge \sigma(\text{vol}_{\perp}) \quad (\text{B3})$$

As discussed in the main text,  $\mathfrak{r} : \mathcal{M}_6 \rightarrow \mathbb{C}$  parametrizes the supersymmetry-breaking, the tangent space  $T_*\mathcal{M}_6$  can be decomposed as  $T_*\mathcal{M}_6 = T_*\Pi \oplus T_*\Pi^{\perp}$  and  $R \in \bigwedge T^*\Pi$ . Recalling that the degree of a polyform is the form-degree of its highest-degree form, we note that

$$|\Psi| = \deg(\Psi) \pmod{2} \quad (\text{B4})$$

The Bianchi identities remain  $dH = 0$  and  $d_H F = -j$ . In the supersymmetric case ( $\mathfrak{r} = 0$ ) any solution to the above equations also solves the full equations of motion. However, for  $\mathfrak{r} \neq 0$ , one also has to impose

$$d[e^{4A-\Phi} \langle \text{Im } (\mathfrak{r}^* \Psi_2), 3\text{Re } \Psi_1 + \frac{1}{2} (-1)^{|\Psi_2|} \Lambda^{mn} \gamma_m \text{Re } \Psi_1 \gamma_n \rangle_3] = 0 \quad (\text{B5})$$

and

$$\text{Im} \{ \langle g_{k(m} dy^k \wedge \imath_n), d_H[e^{A-\Phi} \mathfrak{r}^* (3\text{Re } \Psi_1 + \frac{1}{2} (-1)^{|\Psi_2|} \Lambda^{kr} \gamma_k \text{Re } \Psi_1 \gamma_r)] \rangle \} = 0 \quad (\text{B6})$$

with

$$\Lambda = 1_{\perp} - (g|_{\Pi} + R)^{-1} (g|_{\Pi} - R) \quad (\text{B7})$$

and

$$\gamma_m \omega = (\imath_m + g_{mn} dy^n \wedge) \omega \quad \omega \gamma_m = (-1)^{|\omega|+1} (\imath_m - g_{mn} dy^n \wedge) \omega \quad (\text{B8})$$

For  $R = 0$ , the above simplify considerably due to the identity

$$3\text{Re } \Psi_1 - \frac{1}{2} \Lambda^{mn} \gamma_m \text{Re } \Psi_1 \gamma_n = 4(\text{Re } \Psi_1 - \text{vol}_{\Pi}) \quad (\text{B9})$$

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